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Cosmology and Celestial Mechanics

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The similarities between cosmology and celestial mechanics are discussed from the scientific and historical points of view and the scientific aims of these two fields are compared. Newton's and Poincaré's contributions to celestial mechanics, dynamics, and cosmology are presented. The recently established instability of triple stellar configurations is discussed to relate results of this classical, nonintegrable problem of celestial mechanics to cosmology and to offer an example for "order out of chaos." It is shown that the presently emphasized reasons for limited predictability in dynamical systems are closely related to some of the existing basic difficulties in cosmology.

1. SIMILARITIES IN THE HISTORICAL DEVELOPMENTS

The seeking of observational evidence concerning the geocentric versus the heliocentric views led Copernicus, Kepler, and Galileo to the selection of the heliocentric view. Three hundred years later it is realized that the earth is a medium-size planet orbiting around an average star in our ordinary spiral galaxy, which is one of the 10^{12} galaxies of the observable universe. It should be noted that, with very few exceptions, the generally accepted view prior to Copernicus was that the earth was the center of the universe.

The similarities between the analysis of the solar system and the various theories concerning cosmology cannot escape the reader. The complete heliocentric theory of the solar system cannot have been established without Newton's law of gravity in spite of observational data which were available a considerable time before. (Note that from Kepler's laws—which could have been established by Tycho Brahe before Kepler—Newton's inverse-square force law can be obtained.)

If we wish to accept the historical similarity between cosmology and the dynamical theory of the solar system, we cannot ignore the possibility

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and the need of some new theories, in addition to the well-known requirement for additional observational data when cosmological theories are evaluated.

The erroneous views of the solar system held by most of the Greek philosopher-scientists (Thales of Miletus, Ptolemy of Alexandria, Archimedes, etc.) have some possibly negative aspects concerning our present knowledge of cosmology, if the previously mentioned similarity is taken to its extreme. On the other hand, we might wish to look at the positive aspects of our knowledge concerning the solar system. While the general problem of the stability of the solar system is still not completely solved, reliable numerical integrations show hierarchical stability, which in turn guarantees Laplacian stability, since the former shows the conservation of the relative positions of the orbits of the planets, while the latter excludes collisions and escapes. The meaningful numerical results about the solar system are limited to 10^7 years at the present time, which might be considered a considerable success if compared to the situation which existed some years ago.

2. NEWTON AND POINCARÉ

The contributions of these giants of celestial mechanics are selected since they both pointed out that gravitational dynamics is not sufficient to treat the problems of cosmology. In fact, they realized that the explanation of the origin of the solar system requires more than gravitational effects. This is remarkable, since they both accomplished significant advances in dynamics. Newton's major contributions were his laws of dynamics and his law of gravity, while one of Poincaré's contributions was along negative lines when he showed that the equations of gravitational dynamics in general are not integrable when the number of the participating bodies is more than two.

Newton's ideas concerning cosmology are probably best represented in his letters to Bentley written in 1692–93 (Turnbull, 1961). He point out that if the space is finite, all matter will fall into the middle, composing a great spherical mass. If the space is infinite, the distributed matter would convene into an infinite number of great masses, scattered at great distances. This is Newton's explanation of how the stars are formed (Weinberg, 1977). There are a number of questions regarding this approach to cosmology and Newton himself attacked some of these.

One of Newton's notes in these letters concerning the nature of gravitation is of considerable importance. He makes a strong point that the statement, "Gravity is innate, essential and inherent to matter," should not be attributed to him since he does not pretend to know what the cause of gravity is.

Once again a similarity might be pointed out by quoting de Sitter's agreement with Newton; "Gravitation has no explanation as other physical phenomena, it has no satisfactory hypotheses" (Shapley, 1960).

Newton wishes to make himself clear concerning gravity, when he writes: "That one body may act upon another at a distance is to me so great an absurdity that no man who has any competent faculty of thinking can ever fall into it. Gravity must be caused by an agent acting according to certain laws but whether this agent be material or immaterial is a question I have left to my readers."

This statement is not unexpected from Newton, since he often separated phenomena which could be explained (i.e., which he could explain) by "natural causes" from those which can be attributed to a "voluntary agent." Newton's comments concerning the origin of the solar system offer good examples of his analysis as combined with the contributions of an "agent." Planetary masses arriving from infinity to the sun require changes in the direction and in the magnitude of their velocities in order to take up their circular planetary orbits. Changes of the direction Newton leaves with the "agent." Concerning the required change in the magnitude, he notes that the circular velocity of mass M_P around the solar mass M_S is

$$V_c = \left(G \frac{M_S + M_P}{R_P} \right)^{1/2}$$

where R_P is the radius of the planetary orbit. Furthermore, if the planet left its location at infinity with zero velocity, its arrival velocity should be

$$V_a = \left(2G \frac{M_S + M_P}{R_P} \right)^{1/2}$$

Since these velocities differ by a factor of $\sqrt{2}$, Newton suggested that the solar mass must double at the planet's arrival. In this way the magnitude of the circular velocity will be the same as the arrival velocity. (Note that this requires that $M_P \ll M_S$, which is of little interest here.) Newton arranged capture by doubling the mass of the sun and by invoking the "agent" to change the direction of the velocity of the incoming planet.

It is remarkable to read Newton's idea about the stability of the solar system. This is one of the still unresolved problems of celestial mechanics, as mentioned before, and Newton's intuition is interesting when he notes that larger planetary masses and/or smaller distances would result in too much perturbations and in (Laplacian) instability of the solar system.

Poincaré's not very flattering remark about cosmology is shared by many workers in the field, even today: "Cosmological hypotheses are numerous and varied, one is born every day, all are equally uncertain, but all are as plausible as the older theories" (Shapley, 1960).

His basic idea concerning the evolution of the universe can be described by "order out of chaos" and by his refusal to accept a static universe.

It is interesting to notice that Poincaré's fundamental contributions to analysis and dynamics do not prejudice his opinion concerning cosmological theories when he states that "more physics is needed, in addition to mechanics and mathematics."

3. INSTABILITY OF TRIPLE SYSTEMS

This part of the paper treats a nonintegrable dynamical system which offers an example for the "order out of chaos" principle. First I describe the problem, which will be followed by its surprising qualitative solution. Then I will relate this classical, but only recently "solved" system to cosmology.

Consider three point masses moving with arbitrary initial conditions under their natural gravitational attraction, we wish to describe their behavior as $t \to \infty$. Poincaré has shown the nonintegrability of this dynamical system; therefore, in speaking about "behavior" or "solution" one means the qualitative aspects (Birkhoff, 1927).

If the total energy of the system is positive, the final outcome of the motion is that at least two of the three mutual distances increase to infinity as $t \to \infty$. This expected result, first proven by Chazy in 1918 (see Chazy, 1929) shows a great similarity to the hyperbolic motion of two bodies with positive energy as well as to various cosmological theories often referred to as the "open universe."

The surprising result is associated wih the case of negative total energy when the outcome is once again unbounded. This result was conjectured by Birkhoff (1927) and it was shown by numerical experiments and by using the Lagrange-Jacobi (1772-1892) equation together with Sundman's (1912) inequality by Agekyan and Szebehely in 1967. [For a concise historical description with detailed references and with through analytical treatment see, for instance, Szebehely (1973).]

In summary, the solution in a qualitative sense of the general problem of three bodies might be described as "explosion" when all three bodies depart to infinity (for positive values of the total energy) or "escape" when two bodies form a binary and the third body escapes to infinity. Escape, as mentioned before, will occur for positive as well as for negative total energies. The similarity between the behavior of problems of two and three bodies for positive total energy fails when the total energy is negative, offering another example when preconceived notions concerning integrable dynamical systems cannot be continued to nonintegrable systems.

It is noted that periodic orbits in the general problem of three bodies are neither densely distributed nor are they stable, while the escape-type orbits form continuous families. The escape usually is preceded by interplays of the three bodies and by ejections, when one of the three bodies departs with elliptic velocity and returns to its companions. (These two types of motion were called by Chazy "bounded.")

The equilibrium configurations are no exceptions, since when the masses are of the same order of magnitude, these equilibrium solutions are unstable and transit into interplay.

The numerical analysis faces serious problems, since escapes for negative total energy are usually preceded by triple close approaches which present limitations to the seemingly omnipotent computer approach. Once again, the combination of analytical and numerical approaches results in satisfactory solutions.

Since escapes for negative total energy require prior triple close approaches, it follows that when such a system is placed into the field of a cluster its behavior will not change. The escaping body will leave behind a binary. During its escape it might be captured for new interplays by another binary configuration, but sooner or later this new triple system will also be reduced to a binary and an escaping third body.

The presently known triple stellar configurations belong to the type of motion known as "revolution" when the binary is surrounded by the orbit of the third body. This condition is unstable when the third body is close to the binary and the motion changes into an interplay and escape. If the orbit of the third body is too far from the binary, the system once again becomes unstable because of perturbations due to the other members of the cluster.

The conclusion is that triple systems change into binary systems as $t \to \infty$. The cosmological consequences might be evaluated using the following numerical values which enter the computations. If the masses of the participating bodies are of the order of magnitude of the sun's mass and the initial displacements are of the order of parsecs, the time of disruption of the triple configuration (escape) is of the order of 10^9 years. This result has been shown for arbitrary initial conditions satisfying the requirements of negative total energy. For positive values of the total energy, the disruption of course takes less time. If these results are applied to triple galaxies or any other triple configurations, the order of magnitude of the time of disruption will have to be scaled, but the principle of inherent instability of triple systems will still be valid.

These results consider only gravitational effects, but are based on qualitative considerations of the existing instabilities. The validity of these results for random initial conditions might be considered as another support of the development described, according to which binary configurations should dominate at all levels. If the unstable triple configurations are looked upon as chaotic systems and the binaries as orderly systems, we might propose another example of "order out of chaos" on a global level (Prigogine and Stengers, 1984).

4. PREDICTIONS IN COSMOLOGY

In this section the basic difficulties in the predictions of the behavior of systems for $t \to \pm\infty$ will be discussed and the recently recognized unpredictability (Lighthill, 1986) in celestial mechanics will be related to Lemaître's formulation of the purpose of cosmology.

According to Lemaître, "cosmogonic theories propose to seek out initial conditions which are ideally simple and from which the present world might have resulted, through the natural interplay of known forces" (Shapley, 1960).

The initial conditions referred to by Lemaître can be obtained in principle from integration of the present situation backward, either to $t \to -\infty$ or $t = 0$. Future predictions would correspond to $t = +\infty$.

By briefly surveying predictability in celestial mechanics, we might see the problems faced by cosmology.

1. Predictions in either direction of time require a reasonably accurate set of initial conditions of the variables which enter the physical situation.
2. The knowledge of the laws of nature governing physical phenomena are required with reasonable certainties. In case these laws change in time, the laws of the changes are also required.
3. Poincaré's nonintegrability theorem concerning gravitational systems with more than two participating bodies shows the nonexistence of analytical, generally valid formulas for predictions.
4. Numerical prediction techniques are limited regarding length of time and validity because of the finite digits these computation can carry.
5. The magnitudes and natures of the standard and structural instabilities of the systems determine the accuracy requirements concerning the physical laws and the initial conditions for meaningful predictability.

The above list is directly applicable to dynamical systems and to the science of celestial mechanics. It can be translated also into the language of any other field. Consider, for instance, item 2, which is significant in Newton's cosmology, which does not require fixed laws of nature with change of time. In this way Newton does not claim predictability with arbitrary accuracy (even for simple dynamical systems), while the Leibniz–Laplace approach assumes fixed laws and rules and perfect predictability, assisted by Laplace's demon (Thompson, 1988).

Another aspect of the dependence of predictability on our knowledge concerning the laws of nature is irreversibility, which claims different laws for different directions of time. At this point item 5 enters, since it is well known that even very simple systems show different stability characteristics depending on the arrow of time. This statement is valid for standard stability (depending on the initial conditions) as well as for the sometimes neglected structural stability (depending on the analytical formulation of the problem and on the values of the physical parameters involved).

The most significant item in the above list, when applied to cosmology, is probably the expected instability of the system, since slight changes of initial conditions or of physical formulations (laws) can result in exponential deviations in relatively short time and in bundles of trajectories filling larger and larger volumes of the phase space.

It is interesting to contemplate in this respect the previously mentioned "order out of chaos" of the general gravitational three-body problem. The qualitative outcome is not sensitive to the initial conditions in spite of the fact that the system is unstable. The longer the time of prediction, the simple the prediction becomes. The irreversibility, in a qualitative sense, is established since the escaping body does not return to its original binary. The only significant disadvantage of this system in treating the similarity is that it involves only gravitational forces, and consequently, its applicability to cosmology is questionable. Even this aspect might be looked upon with a positive point of view if it is remembered that the final escape from the binary always occurs after a triple close approach—when gravitational forces are dominant.

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